Financial Economics: Risk Aversion and Investment Decisions

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Outline

- Risk Aversion and Portfolio Allocation
- Portfolios, Risk Aversion, and Wealth
The portfolio problem

**Setup**

Let’s now put our framework of decision-making under uncertainty to use.

Consider a risk-averse investor with vN-M expected utility who divides his or her initial wealth $Y_0$ into an amount $a$ allocated to a risky asset — say, the stock market — and an amount $Y_0 - a$ allocated to a safe asset — say, a bank account or a government bond.
The portfolio problem

Setup

\[ Y_0 = \text{initial wealth} \]
\[ a = \text{amount allocated to stocks} \]
\[ \tilde{r} = \text{random return on stocks} \]
\[ r_f = \text{risk-free return} \]
\[ \tilde{Y}_1 = \text{final wealth} \]
\[ \tilde{Y}_1 = (1 + r_f)(Y_0 - a) + a(1 + \tilde{r}) \]
\[ = Y_0(1 + r_f) + a(\tilde{r} - r_f) \]
The portfolio problem

**Objective Function**

The investor chooses $a$ to maximize expected utility:

$$\max_a E[u(\tilde{Y}_1)] = \max_a E u[Y_0(1 + r_f) + a(\tilde{r} - r_f)]$$

If the investor is risk-averse, $u$ is concave and the first-order condition for this unconstrained optimization problem, found by differentiating the objective function by the choice variable and equating to zero, is both a necessary and sufficient condition for the value $a^*$ of $a$ that solves the problem.
The portfolio problem

First-order Condition

The investor’s problem is

$$\max_a E[u(Y_0(1 + r_f) + a(\tilde{r} - r_f))]$$

The first-order condition (F.O.C.) is

$$E[u'[Y_0(1 + r_f) + a^*(\tilde{r} - r_f)](\tilde{r} - r_f)] = 0$$

Note: we are allowing the investor to sell stocks short ($a^* < 0$) or to buy stocks on margin ($a^* > Y_0$) if he or she desires.
The portfolio problem

Risk Aversion and Portfolio Allocation

Theorem 5.1 If the Bernoulli utility function $u$ is increasing and concave, then

- $a^* > 0$ if and only if $E(\tilde{r}) > r_f$
- $a^* = 0$ if and only if $E(\tilde{r}) = r_f$
- $a^* < 0$ if and only if $E(\tilde{r}) < r_f$

Thus, a risk-averse investor will always allocate at least some funds to the stock market if the expected return on stocks exceeds the risk-free rate.
The portfolio problem

Proof

To prove the theorem, let

\[
W(a) = E[u'[Y_0(1 + r_f) + a(\bar{r} - r_f)](\bar{r} - r_f)]
\]

so that the investor’s first-order condition can be written more compactly as

\[
W(a^*) = 0
\]

it follows that

\[
W'(a) = E[u''[Y_0(1 + r_f) + a(\bar{r} - r_f)](\bar{r} - r_f)^2] < 0
\]

since \( u \) is concave. This means that \( W \) is a decreasing function of \( a \).
Finally, with

\[ W(a) = E[u'[Y_0(1 + r_f) + a(\tilde{r} - r_f)](\tilde{r} - r_f)] \]

\[ W(0) = E[u'[Y_0(1 + r_f)](\tilde{r} - r_f)] \]

\[ = u'[Y_0(1 + r_f)]E(\tilde{r} - r_f) \]

\[ = u'[Y_0(1 + r_f)][E(\tilde{r}) - r_f] \]

Since \( u \) is increasing, this means that \( W(0) \) has the same sign as \( E(\tilde{r}) - r_f \).
We now know that:

- $W(a)$ is a decreasing function
- $W(0)$ has the same sign as $E(\tilde{r}) - r_f$.
- $W(a^*) = 0$
The portfolio problem

Proof

\[ E(\tilde{r}) - r_f > 0 \text{ implies } W(0) > 0, \text{ and since } W \text{ is decreasing, } W(a^*) = 0 \text{ implies } a^* > 0. \text{ Since } W(0) \text{ has the same sign as } E(\tilde{r}) - r_f, \text{ then } E(\tilde{r}) - r_f > 0 \]
Proof

\[ W(a) \]

\[ a^* \]

\[ W(0) \]

\[ Er - r_f < 0 \]
The portfolio problem

**Proof**

\[ W(0) = 0 \]

\[ \bar{a} \]

\[ Er - r_f = 0 \]
Example

suppose \( u(Y) = \ln(Y) \), then \( u'(Y) = 1/Y \), assume that stock returns can either be good or bad:

\[
\tilde{r} = \begin{cases} 
  r_G & \text{with probability } \pi \\
  r_B & \text{with probability } 1 - \pi
\end{cases}
\]

where \( r_G > r_f > r_B \) defines the “good” and “bad” states and

\[
E(\tilde{r}) = \pi r_G + (1 - \pi) r_B > r_f
\]

so that \( E(\tilde{r}) > r_f \) and the investor will choose \( a^* > 0 \)
The first-order condition is

$$E[u'[Y_0(1 + r_f) + a^*(\bar{r} - r_f)](\bar{r} - r_f)] = 0$$

specializes to

$$\frac{\pi(r_G - r_f)}{Y_0(1 + r_f) + a^*(r_G - r_f)} + \frac{(1 - \pi)(r_B - r_f)}{Y_0(1 + r_f) + a^*(r_B - r_f)} = 0$$

implies

$$a^*\frac{1}{Y_0} = -(1 + r_f)[\pi(r_G - r_f) + (1 - \pi)(r_B - r_f)]\frac{1}{(r_G - r_f)(r_B - r_f)}$$

which is positive since $r_G > r_f > r_B$ and

$$E(\bar{r}) - r_f = \pi(r_G - r_f) + (1 - \pi)(r_B - r_f) \geq 0$$
Example

\[
a^* \frac{1}{Y_0} = - \frac{(1 + r_f)(E(\tilde{r}) - r_f)}{(r_G - r_f)(r_B - r_f)}
\]

In this case, \( a^* \)

- Rises proportionally with \( Y_0 \)
- Increases as \( E(\tilde{r}) - r_f \) rises
- Falls as \( r_G \) and \( r_B \) move farther away from \( r_f \), holding \( E(\tilde{r}) \) constant; that is, in response to a mean preserving spread.
Example

\[ a^* = \frac{- (1 + r_f) (E(\tilde{r}) - r_f)}{(r_G - r_f)(r_B - r_f)} \]

<table>
<thead>
<tr>
<th>( r_f )</th>
<th>( r_G )</th>
<th>( r_B )</th>
<th>( \pi )</th>
<th>( E(\tilde{r}) )</th>
<th>( \frac{a^*}{Y_0} )</th>
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The fraction of initial wealth allocated to stocks rises when stocks become less risky or pay higher expected returns.
Before moving on, return to the general problem

$$\max_a E u[Y_0(1 + r_f) + a(\tilde{r} - r_f)]$$

but assume now that the investor is risk-neutral, with

$$u(Y) = \alpha Y + \beta$$

and $\alpha > 0$, so that more wealth is preferred to less. The risk-neutral investor solves

$$\max_a E \alpha[Y_0(1 + r_f) + a(\tilde{r} - r_f)] + \beta$$

$$= \max_a \alpha Y_0(1 + r_f) + a[E(\tilde{r}) - r_f] + \beta$$

So long as $E(\tilde{r}) - r_f > 0$, the risk-neutral investor will choose $a^*$ to be as large as possible, borrowing as much as he or she is allowed to in order to buy more stocks on margin.
The previous examples call out for a more detailed analysis of how optimal portfolio allocation decisions, summarized by the value of $a^*$ that solves

$$\max_a E u[Y_0(1 + r_f) + a(\bar{r} - r_f)]$$

are influenced by the investor’s degree of risk aversion and his or her level of wealth.
Portfolios, Risk Aversion, and Wealth


**Theorem**

**Theorem 5.2** Consider two investors, \( i = 1 \) and \( i = 2 \), and suppose that for all wealth levels \( Y \),

\[
R_A^1(Y) > R_A^2(Y)
\]

where \( R_A^i(Y) \) is investor \( i \)’s coefficient of absolute risk aversion. Then

\[
a_1^*(Y) < a_2^*(Y)
\]

where \( a_i^*(Y) \) is amount allocated by investor \( i \) to stocks when he or she has initial wealth \( Y \).
Portfolios, Risk Aversion, and Wealth

Recall that the coefficients of absolute and relative risk aversion are

\[ R_A(Y) = -\frac{u''(Y)}{u'(Y)} \quad \text{and} \quad R_R(Y) = -\frac{Yu''(Y)}{u'(Y)} \]

Thus

\[ R_A^1(Y) > R_A^2(Y) \quad \text{i.e.,} \quad -\frac{u''_1(Y)}{u'_1(Y)} > -\frac{u''_2(Y)}{u'_2(Y)} \]

implies

\[ -\frac{Yu''_1(Y)}{u'_1(Y)} > -\frac{Yu''_2(Y)}{u'_2(Y)} \quad \text{or} \quad R_R^1(Y) > R_R^2(Y) \]
Portfolios, Risk Aversion, and Wealth

Arrow’s result applies equally well to relative risk aversion:

**Theorem**

**Theorem 5.3** Consider two investors, \( i = 1 \) and \( i = 2 \), and suppose that for all wealth levels \( Y \),

\[
R^1_R(Y) > R^2_R(Y)
\]

where \( R^i_R(Y) \) is investor \( i \)'s coefficient of relative risk aversion. Then

\[
a^*_1(Y) < a^*_2(Y)
\]

where \( a^*_i(Y) \) is amount allocated by investor \( i \) to stocks when he or she has initial wealth \( Y \).
Example with CRRA utility FN

Let’s test Arrow’s proposition out, by generalizing our previous example with logarithmic utility to the case where

\[ u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma} \]

with \( \gamma > 0 \). For this Bernoulli utility function, the coefficient of relative risk aversion is constant and equal to \( \gamma \).
Example with CRRA utility $FN$

Hence

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1-\gamma}, \quad \text{thus} \quad u'(Y) = Y^{-\gamma}$$

and stock returns can either be good or bad

$$\tilde{r} = \begin{cases} r_G & \text{with probability } \pi \\ r_B & \text{with probability } 1-\pi \end{cases}$$

where $r_G > r_f > r_B$ defines the “good” and “bad” states and

$$E(\tilde{r}) = \pi r_G + (1-\pi) r_B > r_f$$

so that $E(\tilde{r}) > r_f$ and the investor will choose $a^* > 0$. 
Example with CRRA utility FN

With CRRA utility and two states for $\tilde{r}$, the F.O.C.

$$E[u'[Y_0(1 + r_f) + a^*(\tilde{r} - r_f)](\tilde{r} - r_f)] = 0$$

specializes to

$$\frac{\pi(r_G - r_f)}{[Y_0(1 + r_f) + a^*(r_G - r_f)]^\gamma} + \frac{(1 - \pi)(r_B - r_f)}{[Y_0(1 + r_f) + a^*(r_B - r_f)]^\gamma} = 0$$
Example with CRRA utility FN

\[ a^* = \frac{(1 + r_f)\left(\pi(r_G - r_f)\right)^{1/\gamma} - \left((1 - \pi)(r_f - r_B)\right)^{1/\gamma}}{(r_G - r_f)\left((1 - \pi)(r_B - r_f)\right)^{1/\gamma} - (r_B - r_f)\left(\pi(r_G - r_f)\right)^{1/\gamma}} \]

<table>
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<th>( r_G )</th>
<th>( r_B )</th>
<th>( \pi )</th>
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<th>( \frac{a^*}{Y_0} )</th>
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</table>

Consistent with Arrow’s theorem, higher coefficients of relative risk aversion are associated with smaller values of \( a^* \).
Example

Portfolios, Risk Aversion, and Wealth

\[
\frac{a^*}{Y_0} = \frac{(1 + r_f)[(\pi(r_G - r_f)]^{1/\gamma} - [(1 - \pi)(r_f - r_B)]^{1/\gamma})}{(r_G - r_f)[(1 - \pi)(r_B - r_f)]^{1/\gamma} - (r_B - r_f)[\pi(r_G - r_f)]^{1/\gamma}}
\]

Note that with constant relative risk aversion, \( a^* \) rises proportionally with wealth.

Two additional theorems, also proven by Arrow, tell us more about the relationship between \( a^* \) and wealth.
Theorem

Theorem 5.4 Let $a^*(Y_0)$ be the solution to

$$\max_a Eu[Y_0(1 + r_f) + a(\tilde{r} - r_f)]$$

If $u(Y)$ is such that

(a) $R'_A(Y) < 0$ then $\frac{da^*(Y_0)}{dY_0} > 0$

(b) $R'_A(Y) = 0$ then $\frac{da^*(Y_0)}{dY_0} = 0$

(c) $R'_A(Y) > 0$ then $\frac{da^*(Y_0)}{dY_0} < 0$
Portfolios, Risk Aversion, and Wealth

Part (a)

\[ R'_A(Y) < 0 \quad \text{then} \quad \frac{da^*(Y_0)}{dY_0} > 0 \]

describes the “normal” case where absolute risk aversion falls as wealth rises.

In this case, wealthier individuals allocate more wealth to stocks.
Portfolios, Risk Aversion, and Wealth

Part (b)

\[ R'_A(Y) = 0 \quad \text{then} \quad \frac{da^*(Y_0)}{dY_0} = 0 \]

means that investors with constant absolute risk aversion

\[ u(Y) = -\frac{1}{\nu} e^{-\nu Y} \]

allocate a constant amount of wealth to stocks.

... so a CARA investor finds a bet of the ideal size and sticks with it, even when income increases.
Portfolios, Risk Aversion, and Wealth

Part (c)

$$R'_A(Y) > 0 \text{ then } \frac{da^*(Y_0)}{dY_0} < 0$$

describes the case where absolute risk aversion rises as wealth rises.

The implication that wealthier individuals allocate less wealth to stocks makes this case seem less plausible.
Consistent with our results with CRRA utility, the next result relates changes in relative risk aversion to changes in the proportion of wealth allocated to stocks. Define the **elasticity** of the function \( a^*(Y_0) \) as

\[
\eta = \frac{d(\ln a^*(Y_0))}{d(\ln Y_0)} = \frac{Y_0}{a^*(Y_0)} \frac{d(a^*(Y_0))}{dY_0}
\]

The elasticity measures the percentage change in \( a^*(Y_0) \) brought about by a percentage-point change in \( Y_0 \).
Portfolios, Risk Aversion, and Wealth

Theorem

**Theorem 5.5** Let \( a^* (Y_0) \) be the solution to

\[
\max_a E u \left[ Y_0 (1 + r_f) + a (\tilde{r} - r_f) \right]
\]

If \( u(Y) \) is such that

(a) \( R'_R(Y) < 0 \) then \( \eta > 1 \)
(b) \( R'_R(Y) = 0 \) then \( \eta = 1 \)
(c) \( R'_R(Y) > 0 \) then \( \eta < 1 \)

The theorem confirms what we know about CRRA utility: it implies that \( a^* \) rises proportionally with \( Y_0 \).

But the theorem extends the results to the cases of decreasing and increasing relative risk aversion.
Portfolios, Risk Aversion, and Wealth

With CRRA

\[ \frac{a^*}{Y_0} = K \]

where

\[ K = \frac{(1 + r_f)([\pi(r_G - r_f)]^{1/\gamma} - [(1 - \pi)(r_f - r_B)]^{1/\gamma})}{(r_G - r_f)[(1 - \pi)(r_B - r_f)]^{1/\gamma} - (r_B - r_f)[\pi(r_G - r_f)]^{1/\gamma}} \]

Hence

\[ \ln(a^*(Y_0)) = \ln(K) + \ln(Y_0) \]

\[ \eta = \frac{d(\ln a^*(Y_0))}{d(\ln Y_0)} = 1 \]
Summary

<table>
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<th>relative risk aversion v.s. relative wealth</th>
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<tbody>
<tr>
<td>absolute value $a^*$</td>
</tr>
<tr>
<td>excess return</td>
</tr>
<tr>
<td>absolute risk aversion</td>
</tr>
<tr>
<td>relative risk aversion</td>
</tr>
</tbody>
</table>
Risk Aversion and Saving Behavior

So far, we’ve assumed that investors only receive utility from the terminal value of their wealth, and asked how they should split their initial wealth—accumulated, presumably, through past saving—across risky and riskless assets in order to maximize the expected utility from terminal wealth.

Now, let’s take the possibly random return on the investor’s portfolio of assets as given, and ask how he or she should optimally determine savings under conditions of uncertainty.
Risk Aversion and Saving Behavior

Suppose there are two periods, \( t = 0 \) and \( t = 1 \), and let

- \( Y_0 = \) initial wealth
- \( s = \) amount saved in period \( t=0 \)
- \( c_0 = Y_0 - s = \) amount consumed in period \( t=0 \)
- \( \tilde{R} = 1 + \tilde{r} = \) random, gross return on savings
- \( \tilde{c}_1 = s\tilde{R} = \) amount consumed in period \( t=1 \)

Suppose also that the investor has vN-M expected utility over consumption during periods \( t = 0 \) and \( t = 1 \) given by

\[
u(c_0) + \beta E[u(\tilde{c}_1)] = u(Y_0 - s) + \beta E[u(s\tilde{R})]
\]

where the discount factor \( \beta \) is a measure of patience.
Risk Aversion and Saving Behavior

The solution to the investor’s saving problem

$$\max_s u(Y_0 - s) + \beta E[u(s\tilde{R})]$$

is described by the first-order condition

$$-u'(Y_0 - s^*) + \beta E[u'(s^*\tilde{R})\tilde{R}] = 0$$

or

$$u'(Y_0 - s^*) = \beta E[u'(s^*\tilde{R})\tilde{R}]$$
Risk Aversion and Saving Behavior

\[ u'(Y_0 - s^*) = \beta E[u'(s^* \tilde{R})\tilde{R}] \]

We can use this optimality condition to investigate how optimal saving \( s^* \) responds to an increase in risk, in the form of a mean preserving spread in the distribution of \( \tilde{R} \). Intuitively, one might expect there to be two offsetting effects:

- The riskier return will make saving less attractive and thereby reduce \( s^* \);
- The riskier return might lead to “precautionary saving” in order to cushion period \( t = 1 \) consumption against the possibility of a bad output and thereby increase \( s^* \).
Risk Aversion and Saving Behavior

\[ u'(Y_0 - s^*) = \beta E[u'(s^* \tilde{R}) \tilde{R}] \]

To see which of these two effects dominates, define

\[ g(\tilde{R}) = u'(s^* \tilde{R}) \tilde{R} \]

so that the right-hand side becomes

\[ \beta E[g(\tilde{R})] \]

Jensen’s inequality will imply that after a mean preserving spread the distribution of \( \tilde{R} \) in this expectation will fall if \( g \) is concave and rise if \( g \) is convex.
Risk Aversion and Saving Behavior

When $g$ is concave, a mean preserving spread in the distribution of $\tilde{R}$ will decrease $E[g(\tilde{R})]$. 
Risk Aversion and Saving Behavior

When $g$ is convex, a mean preserving spread in the distribution of $\tilde{R}$ will increase $E[g(\tilde{R})]$. 
Risk Aversion and Saving Behavior

The definition

\[ g(\tilde{R}) = u'(s^*\tilde{R})\tilde{R} \]

suggests that the concavity or convexity of \( g \) will depend on the sign of the third derivative of \( u \).

The product and chain rules for differentiation imply

\[
\begin{align*}
g'(\tilde{R}) &= u''(s^*\tilde{R})s^*\tilde{R} + u'(s^*\tilde{R}) \\
g''(\tilde{R}) &= u'''(s^*\tilde{R})(s^*)^2\tilde{R} + u''(s^*\tilde{R})s^* + u''(s^*\tilde{R})s^* \\
&= u'''(s^*\tilde{R})(s^*)^2\tilde{R} + 2u''(s^*\tilde{R})s^*
\end{align*}
\]

implies that \( g''(\tilde{R}) \) has the same sign as

\[
u'''(s^*\tilde{R})s^*\tilde{R} + 2u''(s^*\tilde{R})
\]
Risk Aversion and Saving Behavior

To understand precautionary saving behavior, the concept of prudence is defined by Miles Kimball, “Precautionary Saving in the Small and in the Large,” *Econometrica* Vol.58 (January 1990): pp.53-73.

Whereas risk aversion is summarized by the second derivative of the Bernoulli utility function $u$, prudence is summarized by the third derivative of $u$. 
Risk Aversion and Saving Behavior

Kimball defines the coefficient of absolute prudence as

\[ P_A(Y) = -\frac{u'''(Y)}{u''(Y)} \]

and the coefficient of relative prudence as

\[ P_R(Y) = -\frac{Yu'''(Y)}{u''(Y)} \]

thereby extending the analogous measures of absolute and relative risk aversion.
Risk Aversion and Saving Behavior

\[ g''(\tilde{R}) \text{ has the same sign as } \]
\[ u'''(s^*\tilde{R})s^*\tilde{R} + 2u''(s^*\tilde{R}) \]

or
\[ u'''(Y)Y + 2u''(Y) = u''(Y)[\frac{Yu'''(Y)}{u''(Y)} + 2] = u''(Y)[2 - P_R(Y)] \]

\[ g''(\tilde{R}) \text{ is positive if } 2 < P_R(Y); \]
\[ g''(\tilde{R}) \text{ is negative if } 2 > P_R(Y) \]
Risk Aversion and Saving Behavior

Hence, if \( 2 < P_R(Y) \), then \( g''(\tilde{R}) > 0 \). Since \( g \) is convex, a mean preserving spread in the distribution of \( \tilde{R} \) increases the right hand side of the optimality condition

\[
u'(Y_0 - s^*) = \beta E[u'(s^* \tilde{R})\tilde{R}]\]

and \( s^* \) must increase to maintain the equality. The precautionary saving effect dominates if the coefficient of relative prudence exceeds 2.
Conversely, if $2 > P_R(Y)$, then $g''(\tilde{R}) < 0$. Since $g$ is concave, a mean preserving spread in the distribution of $\tilde{R}$ decreases the right hand side of the optimality condition

$$u'(Y_0 - s^*) = \beta E[u'(s^* \tilde{R})\tilde{R}]$$

and $s^*$ must decrease to maintain the equality. The precautionary saving effect dominates if the coefficient of relative prudence is less than 2.
Risk Aversion and Saving Behavior, Example

To apply these results, let’s calculate the coefficient of relative prudence implied by the CRRA utility function

$$u(Y) = \frac{Y^{1-\gamma} - 1}{1-\gamma}$$

where $\gamma > 0$, since $u'(Y) = Y^{-\gamma}$

$$u''(Y) = -\gamma Y^{-\gamma-1}$$

$$u'''(Y) = \gamma(\gamma + 1)Y^{-\gamma-2}$$

imply

$$P_R(Y) = -\frac{Y u'''(Y)}{u''(Y)} = \frac{Y \gamma(\gamma + 1)Y^{-\gamma-2}}{\gamma Y^{-\gamma-1}} = \gamma + 1$$
Risk Aversion and Saving Behavior, Example

Hence, the CRRA utility function

\[
u(Y) = \frac{Y^{1-\gamma} - 1}{1 - \gamma}
\]

implies both a constant coefficient of relative risk aversion equal to \( \gamma \) and a constant coefficient of relative prudence equal to \( \gamma + 1 \).

If \( \gamma > 1 \), saving rises in response to a mean preserving spread in the distribution of \( \tilde{R} \). When \( \gamma < 1 \), saving falls. In the special case \( \gamma = 1 \) of logarithmic utility, saving is unaffected.